

## A NOTE ON ROGER'S FINE IDENTITY

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**Abstract:** In this paper, making use of Rogers-Fine identity and certain known results, interesting and useful results have been established.

**Keywords and Phrases:** Rogers-Fine identity, Continued fraction, Lambert series, Generalized Lambert series.

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### 1. Introduction and Definitions

For  $q$  real or complex  $|q| < 1$ , a basic hypergeometric series is given by

$${}_r\Phi_s \left[ \begin{matrix} a_1, a_2, \dots, a_r; q; z \\ b_1, b_2, \dots, b_s; q^\lambda \end{matrix} \right] = \sum_{n=0}^{\infty} \frac{[a_1, a_2, \dots, a_r; q]_n z^n}{[q, b_1, b_2, \dots, b_s; q]_n} q^{\lambda n(n-1)/2}, \quad (1.1)$$

where  $[a_1, a_2, \dots, a_r; q]_n = [a_1; q]_n [a_2; q]_n \dots [a_r; q]_n$ ,

$$[a; q]_n = \begin{cases} 1, & \text{if } n = 0, \\ (1-a)(1-aq)(1-aq^2)\dots(1-aq^{n-1}), & \text{if } n = 1, 2, 3, \dots \end{cases}$$

and

$$[a; q]_{\infty} = \prod_{r=0}^{\infty} (1 - aq^r). \quad (1.2)$$